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Understanding the influence of distractors on workload capacity

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HIGHLIGHTS

- We highlight the link between workload capacity and mental architecture.
- We show how including distractors may change the predicted minimum time.
- We show how this change to the minimum time alters the capacity coefficient.
- We show how to recover the diagnosticity of the capacity coefficient by varying distractor discriminability.
- We term this new measure resilience to emphasize the inclusion of distractors.

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ABSTRACT

In this paper, we analyze the *workload capacity* of information processing of multidimensional perceptual stimuli. Capacity, which describes how the processing rate of the system changes as the number of stimulus dimensions or attributes is increased, is an important property of information processing systems. Inferences based on one measure of capacity, the capacity coefficient (Townsend and Nozawa, 1995), are typically computed by comparing the processing of *single targets*, which provide a measure of the baseline processing time of the system, to the processing of a *double target*. The single targets are typically assumed to be presented alone without any irrelevant distracting information. In this paper, we derive new capacity predictions for situations when distractor information is present. This extension reveals that, with distractors, the value of the capacity coefficient no longer provides unique diagnostic information about the underlying processing system. We further show how to rectify this situation by contrasting distractors of different discriminability.

People often need to make quick and accurate decisions in complex environments. Performance may be impaired due to the increase in the number of to-be-processed signals (hereafter *load*), and the presence of distracting signals (*distractors*). The current paper develops a framework for assessing the effects of distractors on the measurement of human performance under varying load conditions. The capacity coefficient (Townsend & Nozawa, 1995) is a measure of human information processing with increased workload, calculated by comparing the time it takes to process multiple targets to the time it takes to process each target in isolation. It is operationalized by comparing the processing of a given system (or architecture) to a well-studied benchmark—an independent-channel parallel model (which we introduce in more detail shortly). In their seminal work Townsend and Nozawa measured the capacity of the system in a *redundant target* visual-detection task in which participants could be presented with a display containing two luminance dots, a single dot on the right hand side, a single dot on the left hand side, or no dots at all. Their task was to detect the presence of any target (i.e., dot on the right, left, or both locations) and press a "yes, target present" key; otherwise, participates pressed a different key or simply withheld their response. A similar task could have been to detect the target letter "X", where XX is the double-target condition and is ultimately compared to detection latencies of a single target—X alone, either on the right on left location.

The above is a canonical example of a 'distractor-free' detection task, where signals could appear or not, but if they do appear they are necessarily targets. In another variant of this task

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(discrimination, rather than detection), participants could again be presented with two target signals or just one (or none). In contrast to the 'distractor-free' example, however, other signals that are not the to-be-detected target could appear as well. For example, double target displays would again be XX, but single-target displays would be XO or OX and processing would be required to be focused only on the relevant target information. Thus, some displays could be accompanied by a distractor item that is irrelevant to the decision. This creates a challenge for the calculation and interpretation of the capacity coefficient: moving from one to two (or more) target-signals incurs more than just a change in load; it is also accompanied by changes in the distractor information. To calculate capacity one needs to take into account not only how efficiently the system processes two targets as opposed to just one target, but also potential effects due to the presence of distractors. For instance, superior performance with two targets (XX) vs. one target-one distractor display (XO) could mark efficient processing in the former condition, but could also be a consequence of slow-down in the latter due to the presence of the unhelpful (and possibly harmful) distractor item.

To date, research using the capacity coefficient has focused primarily on cognitive tasks in which the target is presented without distractors. In those studies that have used distractors as part of their design (e.g., Ben-David, Eidels, & Donkin, 2014) the capacity coefficient allowed only limited interpretation due to the competition between effects of load and distraction. The purpose of the current paper is to extend the applicability of the Townsend and Nozawa's (1995) capacity coefficient to cognitive tasks in which distracting information could be present along with target information.

Such an extension would expand the range of cognitive tasks that can be studied using this informative statistic to domains involving distractors. In particular, it would allow one to consider the role of distractor information (additional items or additional dimensions within an item) available in the standard designs of many psychological tasks, such as (but not limited to) categorization (Fific, Little, & Nosofsky, 2010; Little, Nosofsky, & Denton, 2011; Little, Nosofsky, Donkin, & Denton, 2013), recognition memory (Nosofsky, Little, Donkin, & Fific, 2011; Townsend & Fific, 2004), detection (Feintuch & Cohen, 2002; Mordkoff & Yantis, 1993), discrimination (Donkin, Little, & Houpt, 2014), and visual search (Ben-David & Algom, 2009; Fifić, Townsend, & Eidels, 2008; Thornton & Gilden, 2007). Furthermore, tasks that examine stimulus–response congruence, such as the Stroop (Stroop, 1935), Simon (Proctor & Vu, 2006; Simon & Rudell, 1967), and flanker (Eriksen & Eriksen, 1974) tasks, manipulate and measure the effects of conflicting sources of information in a way we can analyze using the machinery developed in this paper and that was not previously available. Like the initial development of the capacity coefficient (Townsend & Nozawa, 1995), our extension is derived for regimes involving near error-free performance.

The logic of our extension is illustrated with a simple case of one target and one distractor, as shown in Fig. 1. The figure illustrates the difference between distractor-free and distractor-present tasks. The left and middle panels of Fig. 1 show two variants of the (distractor-free) redundant-target detection task. The left-hand side panel depicts a task with an OR decision rule, where an observer should respond YES if she detects a target in the left *or* the right locations or both. The middle panel illustrates an AND decision rule, where an observer should respond YES only if targets appear on both the left *and* right locations. By contrast, the task depicted on the right-hand side panel requires discrimination of a target (low luminance dot) from distractors (high luminance dot).[1]

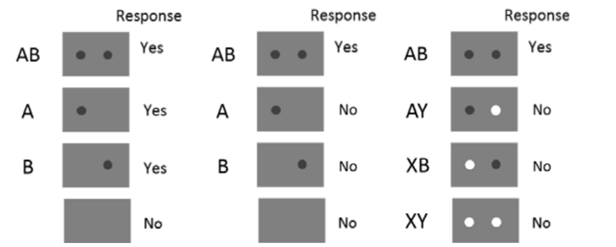

Fig. 1. Examples of detection (panels A and B) and discrimination (panel C) tasks.

Since the single- and null-target displays contain a non-target dot possibly alongside with the target information, accurate responding requires that the high-luminance target be discriminated from the low-luminance distractor. In the case where capacity is not unlimited, then processing the distractor information may occupy non-negligible processing time. Since the capacity coefficient is an RT-based measure, the presence of distractors can alter its value.

The influence of distractors on cognitive operations cannot be investigated separately from the role of *processing architectures* underlying those operations. A cognitive architecture defines how processes underlying cognitive operations are organized, in terms of processing order (serial, parallel), and stopping rule (whether it is possible to stop after a limited amount of information has been processed – self-terminating – or only after all information had been processed — exhaustive). Joint consideration of mental architectures and distractor information is critical in assaying the capacity function. For illustration, assume that a participant is using the *serial exhaustive system* to search for certain target items. In such a system, any two items (or more) are processed in a sequential fashion, and the processing is completed only when *both* are processed. Distractor items in such system will be mandatorily processed along with targets, since the cognitive system cannot stop upon the detection of the target. In contrast, a *serial self-terminating system* can make a decision as soon as a target was found, and before completing the processing of distractor information. Thus, two serial systems with different stopping rules will be affected by the presence of distractor information in different ways. The capacity coefficient statistic is sensitive to these differences as is revealed by the formal definition of the capacity coefficient provided in the next section.

Intuitively, the coefficient is expressed as a ratio between performance on the double-target condition and the minimum-time prediction derived from the single-target conditions. An unlimited capacity parallel model, which is used as the baseline comparison model, predicts that these quantities should be equivalent; hence, their ratio (the capacity coefficient, $C(t)$) should equal 1 across all observed response times (i.e., $C(t) = 1$). The presence of distractors may affect how quickly single-target trials are processed, and reduce or increase the minimum time predicted from the target + distractor trials. This, in turn, affects the inferences that one can derive from the capacity coefficient. For example, in the standard, distractor-free case (see Fig. 1, left and middle panels), limited capacity models predict $C(t) < 1$; however, the same limited capacity models can predict $C(t) = 1$ or $C(t) > 1$ when distractors are present in the display. Likewise, supercapacity models (such as coactive or facilitatory interactive models, e.g., Eidels, Houpt, Altieri, Pei, & Townsend, 2011), which exhibit double-target processing that is faster than the benchmark minimum-time prediction of independent single targets (i.e.,

[1] It should be noted that the high-luminance white dot is not "informationless" but provides positive evidence for a NO response (or, equivalently, negative evidence for a YES response). Consequently, whether or not an AND or an OR rule is applied depends on whether the observer frames the task as detecting two low luminance black dots on the left and the right or detecting a white dot on the left or the right.

$C(t) > 1$), may predict double-target processing rates that are even faster than the derived minimum time when distractors are present (i.e., further increasing the inferred capacity of the system). To foreshadow, we will show below that the influence of distractors is systematic and predictable. By varying the processing rate of the distractors via experimental manipulations of their discriminability, one can construct a novel contrast, between one measure based on the slow-processed distractor and a second measure based on the fast-processed distractor. This contrast allows recovery of information about the architecture of the underlying processing system.

In the following, we provide a formal definition of the capacity coefficient (Townsend & Nozawa, 1995; Townsend & Wenger, 2004). We then show how measures of capacity change in the presence of distractor information for several important processing architectures. Finally, we show that by contrasting capacity with distractors of different discriminability, a novel diagnostic measure, the *Resilience Difference* function, $R_{diff}(t)$, allows one to accurately diagnose processing architecture.

The capacity coefficient, $C(t)$

The workload capacity coefficient ($C(t)$; Townsend & Nozawa, 1995) is an RT-based measure of performance relative to what one would expect under a standard benchmark model—the Unlimited-Capacity Independent-channel Parallel (UCIP) model. This benchmark model predicts a capacity of 1 for all t for the case when one must respond using an OR decision rule (e.g., the OR task shown in Fig. 1). That is:

$$C_{OR}(t) = \frac{-\ln [S_{12}(t)]}{-\ln [S_1(t) \times S_2(t)]} \tag{1}$$

where $S_{12}(t)$ is the survivor function of response time when both targets are present, and $S_1(t)$ and $S_2(t)$ are the survivor functions when only target 1 or target 2 is present. Townsend and Wenger (2004) derived a comparable capacity index for an AND decision rule (e.g., the AND task shown in the middle panel of Fig. 1),

$$C_{AND}(t) = \frac{\ln [F_1(t) \times F_2(t)]}{\ln [F_{12}(t)]} \tag{2}$$

where $F(t)$ is the cumulative distribution function.[2] Under both measures, $C(t)$ of 1 for all t indicates unlimited capacity, $C(t)$ less than 1 indicates limited capacity, and $C(t)$ greater than 1 indicates supercapacity.

$C(t)$ provides a way to understand the capacity of a system as well as the way information is processed by that system. For example, under the assumption of context invariance (e.g., that the processing time of channel 1 is unaffected by the presence or absence of channel 2), serial models, which process each target one at a time, predict limited capacity in an OR task because increasing the number of items to be processed slows down the overall processing time of the system (e.g., Townsend & Ashby, 1983). This occurs regardless of whether the serial model processes both targets *exhaustively* or terminates processing as soon as a target is detected (e.g., *self-terminating* processing). In both cases, the redundant target processing time will be slower than the derived minimum time of the single targets. The same is true for parallel exhaustive models in an OR task. Alternatively, an independent parallel model, which processes information simultaneously but terminates processing as soon as a target is detected, predicts unlimited capacity because increasing the number of items to-be-processed does not slow down or speed up the overall processing time. In contrast, coactive models, which pool information across channels, predict redundant processing times which are faster than the derived minimum time of the single targets. As a consequence, we might expect that capacity can be inferred solely from understanding architecture and, to some extent, vice versa.[3]

Fig. 2 shows the $C_{OR}(t)$ and $C_{AND}(t)$ predictions for the serial self-terminating, serial exhaustive, parallel self-terminating, parallel exhaustive, and coactive models. Accurate responding in an AND task requires exhaustive processing; self-terminating responding results in an unacceptably low accuracy. Consequently, we only present the capacity functions for highly accurate responding for the AND case (Fig. 2, bottom row). The OR task naturally calls for minimum-time processing, "respond as soon as you detect target item 1 *or* target items 2". However, participants may be unnecessarily exhaustive, and hence slow, but will not suffer any accuracy decrement.

In the following section, we consider the capacity predictions of each of the pertinent models without distractors. We focus on the aspects that are necessary to extend capacity analysis to cases where distractor information is presented as well. In this initial presentation, we restrict discussion solely to the case where, using the example presented in Fig. 1, the system correctly detects the low luminance (black) target despite the presence of distractors. The focus on correct decision times is a simplifying limitation but one that has several advantages. For one, it allows for the development of novel information-processing measures in a tutorial fashion. Second, it provides a basis for comparison with Townsend and Nozawa's (1995) development of the capacity coefficient, allowing links between our measure and their measure.

$C(t)$ predictions for models without distractors

The capacity predictions of the models we review in this section are presented elsewhere (Townsend & Ashby, 1983; Townsend & Nozawa, 1995), but we collect these results here as it allows for efficient extension once distractors are introduced.

OR task: parallel, independent, self-terminating model without distractors (benchmark model). If processing of both information channels (1 and 2) is parallel, independent, and self-terminating, then the response time of the entire model is determined by the minimum processing time:

$$F_{12}^{parallel}(t) = 1 - ([1 - F_1(t)] \times [1 - F_2(t)]). \tag{3}$$

This equation gives the cumulative distribution function (cdf) for the minimum time distribution. Alternatively, in terms of the survivor functions,

$$S_{12}^{parallel}(t) = S_1(t) \times S_2(t). \tag{4}$$

Substituting Eq. (4) into $C_{OR}(t)$ (Eq. (1)) makes it clear that an independent parallel self-terminating processing model predicts a $C_{OR}(t)$ function that equals 1 for all t, which provides one intuition for why the UCIP model is used as a benchmark.

[2] In the following, we use $f^{parallel}(t)$, $F^{parallel}(t)$, and $S^{parallel}(t)$ when referring specifically to the pdf, cdf, and survivor functions of the parallel model, and we use $f^{serial}(t)$, $F^{serial}(t)$, and $S^{serial}(t)$ when referring to the pdf, cdf, and survivor functions of the serial model. We use the generic $f(t)$, $F(t)$, and $S(t)$ when the distinction between serial and parallel does not matter.

[3] Several extensions to this capacity measure have been developed. First, Townsend and Eidels (2011) developed a framework, and mathematical formulas to relate several important response-time inequalities with the capacity coefficient, $C(t)$. These inequalities include upper and lower boundaries on unlimited capacity in an OR design (Grice, Canham, & Gwynne, 1984; Miller, 1982), and comparable upper and lower bounds for the AND case (Colonius & Vorberg, 1994). Second, Townsend and Altieri (2012) offered provisions for decoupling accuracy and response-time capacity measures. These measures are beyond the scope of the present paper but are important related developments.

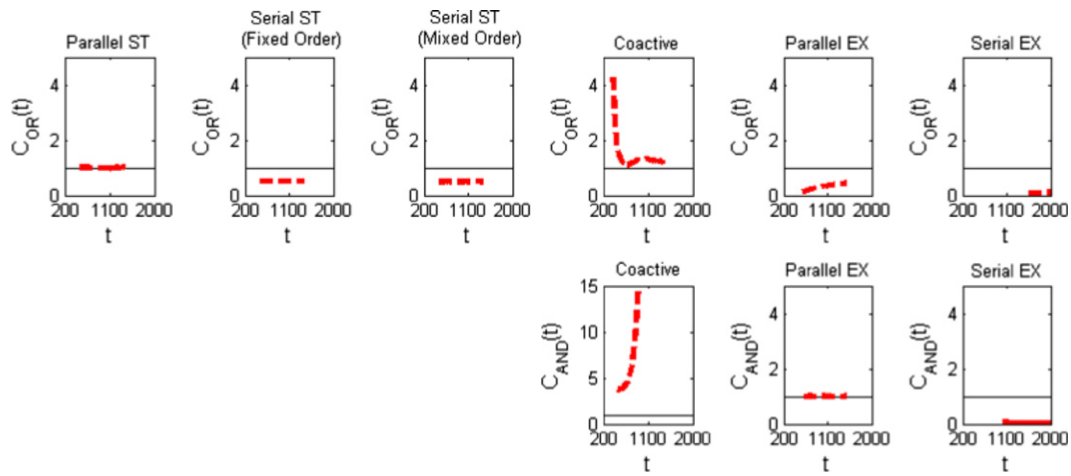

Fig. 2. Capacity predictions for each of the models in the OR task (top panels) and the AND task (bottom panels). Each panel shows the results of a linear ballistic accumulator (Brown & Heathcote, 2008) simulation. For all simulations, drift rates for a target were set to 0.69. The redundant target drift rate for the coactive model was set to 0.90. The boundaries were set at 0.8, the range of the uniform start point distribution was 0.5, and the non-decision time was set to 0. Error accumulator drift rates were one minus the correct accumulator drift rates.

AND task: parallel, independent, exhaustive model without distractors (benchmark model). RT in an AND task is determined by the maximum processing time, which is:

$$F_{12}^{parallel}(t) = F_1(t) \times F_2(t). \tag{5}$$

Again, comparison of Eqs. (5) and $C_{AND}(t)$ equation (Eq. (2)) illustrates that the benchmark UCIP model predicts $C_{AND}(t) = 1$ for all t.

The capacity predictions of the serial and coactive models can be given intuitive content by considering how the survivor function from the double-target condition compares to the product of the survivors of the two single targets (i.e., the minimum-time distribution, Eq. (4)) in the OR condition or the product of the cdfs of the two single targets (i.e., the maximum-time distribution, Eq. (5)) in the AND condition.

OR task: serial, independent, self-terminating model without distractors. In an OR task, a serial self-terminating model predicts that processing will terminate as soon as either target is processed. Let p equal the probability that channel 1 is processed before channel 2 (and similarly $1-p$ is the probability that 2 is processed before 1). Channels 1 and 2 can refer to spatial locations, as in Fig. 1, but more broadly could be any two sources of information (e.g., two modalities, or even two attributes of an object—such as color and shape) for which processing can be scheduled in a principled way. The RT density function for the double target condition in a serial, self-terminating model is therefore a mixture of trials from two types:

$$f_{12}^{serial}(t) = p\,[f_1(t)] + (1-p)\,[f_2(t)]. \tag{6}$$

Integrating this function with respect to t gives the cdf, $F_{12}(t)$, for the double target condition:

$$F_{12}^{serial}(t) = \int_0^t p\,[f_1(t)] + (1-p)\,[f_2(t)]\,dt. \tag{7}$$

Converting the cdf to the survivor function and substituting the serial-model survivor function into Eq. (4) gives:

$$\begin{aligned} S_{12}^{serial}(t) &= 1 - \int_0^t p\,[f_1(t)] + (1-p)\,[f_2(t)]\,dt \\ &> \left(\left[1 - \int_0^t f_1(t)\,dt\right] \times \left[1 - \int_0^t f_2(t)\,dt\right]\right) \\ &= S_1(t) \times S_2(t) \\ &= S_{12}^{parallel}(t) \end{aligned} \tag{8}$$

for a serial, self-terminating model, which implies that the $C_{OR}(t)$ will be less than 1 for all t, indicating limited capacity. In other words, because the serial self-terminating model predicts that the double-target RT distribution will be roughly the same as single target RT distribution (i.e., because in both cases, the model self-terminates after processing one channel), the double-target RT distribution of the serial model will be slower (i.e., the survivor function would be greater) than the minimum-time RT distribution of the single-target conditions from the serial model (i.e., $C(t)$ is less than 1).

AND task: serial, independent, exhaustive model without distractors. In the AND task, given double targets, a serial model must process to completion both targets (i.e., exhaustive processing). Consequently, the RT density for the double AND target is:

$$f_{12}^{serial}(t) = f_1(t) * f_2(t), \tag{9}$$

where $f_1(t) * f_2(t)$ is the convolution of processing-time densities in channels 1 and 2. Integrating the density to obtain the cdf, $F_{12}(t)$, and substituting into Eq. (5) gives:

$$\begin{aligned} F_{12}^{serial}(t) &= \int_0^t f_1(t) * f_2(t)\,dt \\ &< \int_0^t f_1(t)\,dt \times \int_0^t f_2(t)\,dt \\ &= F_1(t) \times F_2(t) \\ &= F_{12}^{parallel}(t), \end{aligned} \tag{10}$$

which again implies limited capacity for all t. In other words, because in the AND task the serial self-terminating model must process both of the targets in the double target condition and the RT is given by the convolution of these targets, the double target RT distribution from the serial model is even slower than the maximum-time distribution predicted under the assumption of the baseline model (i.e., $C(t)$ is less than 1).

Coactive model. A system is coactive if parallel processing channels pool their activation into a single, common decision buffer, rather than making separate decisions on each channel. Pooling evidence in favor of multiple targets into a single buffer means that activation is built up more rapidly with two or more targets (channels) compared with just one. Concomitantly, coactive models have been shown to demonstrate supercapacity for all t (Eidels et al., 2011; Townsend & Eidels, 2011; Townsend & Nozawa, 1995; Townsend & Wenger, 2004). For example, in

Eidels et al. (2011), coactive processing was modeled by summing the evidence rates of the single targets. This pooled processing produced responses to double targets that were faster than the minimum-time predictions of the baseline UCIP model, resulting in $C(t)$ values greater than 1.

From capacity to resilience: model predictions with distractors

The standard approach to measuring workload capacity, outlined above, cannot be used to estimate a system's capacity in the presence of distractors. With distractors, single target displays have information to process in both channels, and hence the capacity measure does not assess workload per se but only the relative rates of processing of the target and distractor information. By relative rates of processing, we mean the processing of two targets relative to the processing of a target and a distractor. As outlined below, the inclusion of distractors will change the behavior of the capacity coefficient such that the serial self-terminating, serial exhaustive, parallel exhaustive, and coactive models no longer make the same predictions for the single targets (i.e., when no distractors are present). It is no longer accurate in such a case to refer to this measure as *workload* capacity, where workload refers to a comparison of the change in processing as the number of different active processing channels increases (e.g., from one in the single target case to two in the double target case). Instead, we propose the term *resilience* to reflect that the function indicates how each model deals with the distracting information. Surprisingly, however, the same functional form shown in Eq. (1) may be used to study resilience; hence, the resilience function for the case when the single targets contain distracting information is:

$$R(t) = \frac{-\ln\left[S_{AB}(t)\right]}{-\ln\left[S_{AY}(t) \times S_{XB}(t)\right]}. \tag{11}$$

Note that we adopt the subscripts A and B to refer to the target information (i.e., task-relevant information that can be used to make a correct decision) and the subscripts X and Y to refer to distracting information (see Fig. 1, right-hand side panel). Proofs of the relevant results, which we summarize below, are provided in the Appendix.

Although the distractor channels can provide useful evidence (e.g., the presence of a distractor suggests there is no target in this location, hence negative evidence for a YES decision), processing a single distractor does not allow one to terminate the response successfully. This is analogous to the original capacity design in which nothing is presented in the non-target location (see Fig. 1, left-hand side and middle panels); however, in that design, the absence of any information may attract only negligible processing for that location. In the present case, the detection of a distractor may attract processing, but the determination that the location contains a distractor only signals that the other channel needs to be processed to determine whether the target information is present. The distinction we make is between identifying the presence of the target when no other item is presented in the other location and discriminating distractor information from target information. In the case where the discrimination between the target and distractor is very easy, then the standard capacity coefficient function is likely still useful (i.e., Eq. (11) becomes Eq. (1)).[4]

OR task: parallel, independent, self-terminating model with distractors. In an OR task, the resilience predictions of the parallel OR model *will not differ* from the predictions of the (ordinary) capacity coefficient because the single target RTs will be based on the finishing time of A and B, which for a UCIP model remains the same regardless of the presence or absence of distractors.[5] That is, a *correct response* to the single target–single distractor trials can only be made after a target is detected; processing a distractor provides no information about the presence or absence of a target in the other location, and, in an independent parallel model, will have no effect on target channel processing. For a serial model, however, the RTs are likely to be affected by the presence of distractors.

OR task: serial, independent, self-terminating model with distractors. The RT density of a serial, self-terminating model for the double target is given by Eq. (6). For the single target–single distractor conditions, however, the RT densities are:

$$f_{AY}^{serial}(t) = p\left[f_A(t)\right] + (1-p)\left[f_Y(t) * f_A(t)\right] \tag{12}$$

and

$$f_{XB}^{serial}(t) = p\left[f_X(t) * f_B(t)\right] + (1-p)\left[f_B(t)\right]. \tag{13}$$

Substituting Eqs. (6), (12) and (13) into Eq. (4) and rearranging gives:

$$\begin{aligned} S_{AB}^{serial}(t) &= 1 - \int_0^t \left(p\left[f_A(t)\right] + (1-p)\left[f_B(t)\right]\right) dt \\ &= \left[1 - \int_0^t \left(p\left[f_A(t)\right] + (1-p)\left[f_Y(t) * f_A(t)\right]\right) dt\right] \\ &\quad \times \left[1 - \int_0^t \left(p\left[f_X(t) * f_B(t)\right] + (1-p)\left[f_B(t)\right]\right) dt\right] \\ &= S_{AY}^{serial}(t) \times S_{XB}^{serial}(t). \end{aligned} \tag{14}$$

If processing occurs in a fixed-order such that channel 1, containing A or X, is always processed before channel 2, containing B or Y (i.e., $p = 1$), then Eq. (14) reduces to:

$$\begin{aligned} S_{A}^{serial}(t) &= 1 - \int_0^t \left(f_A(t)\right) dt \\ &= \left[1 - \int_0^t \left(f_A(t)\right) dt\right] \times \left[1 - \int_0^t \left(f_X(t) * f_B(t)\right) dt\right] \\ &= S_{A}^{serial}(t) \times S_{XB}^{serial}(t). \end{aligned} \tag{15}$$

The negative logarithm of the survivor function is equal to the integrated hazard function, $H(t)$, since $S(t) = \exp(-H(t))$ (Townsend & Nozawa, 1995). Consequently, restating the equation in terms of the survivor functions and taking the *negative* logarithm of both sides gives:

$$-\ln S_{A}^{serial}(t) \leq -\ln S_{A}^{serial}(t) - \ln S_{XB}^{serial}(t). \tag{16}$$

Another way to think about this situation is to consider what happens when the presence of X slows down the single target XB by a large amount. In a fixed-order processing model where channel 1 is processed first, AY will always be faster than XB (assuming that A and B are processed at roughly the same rate),

[4] Even though we limit our discussion to the case when correct decisions are made, it is not the case that high accuracy implies that the distractor will not influence response time. There are many cases where the difficulty of a stimulus has an effect of response time but not on accuracy (see e.g., Fifić et al., 2010; Logan, 1996; Sternberg, 1966; Townsend & Nozawa, 1995).

[5] Certain models (e.g., Leaky Competing Accumulator, LCA, Usher & McClelland, 2001) suggest that distractors may inhibit processing of target items. However, the UCIP model, by definition, precludes cross-channel inhibition. Eidels et al. (2011) show how the SIC function and capacity can be used to differentiate interactive parallel models such as the LCA in the single targets without distractor case. The present work could be extended to examine interactive parallel models; however, we leave that for future research. We expect that cross-channel inhibition or facilitation would have effects on $R(t)$ consistent with those demonstrated by Eidels et al. (2011).

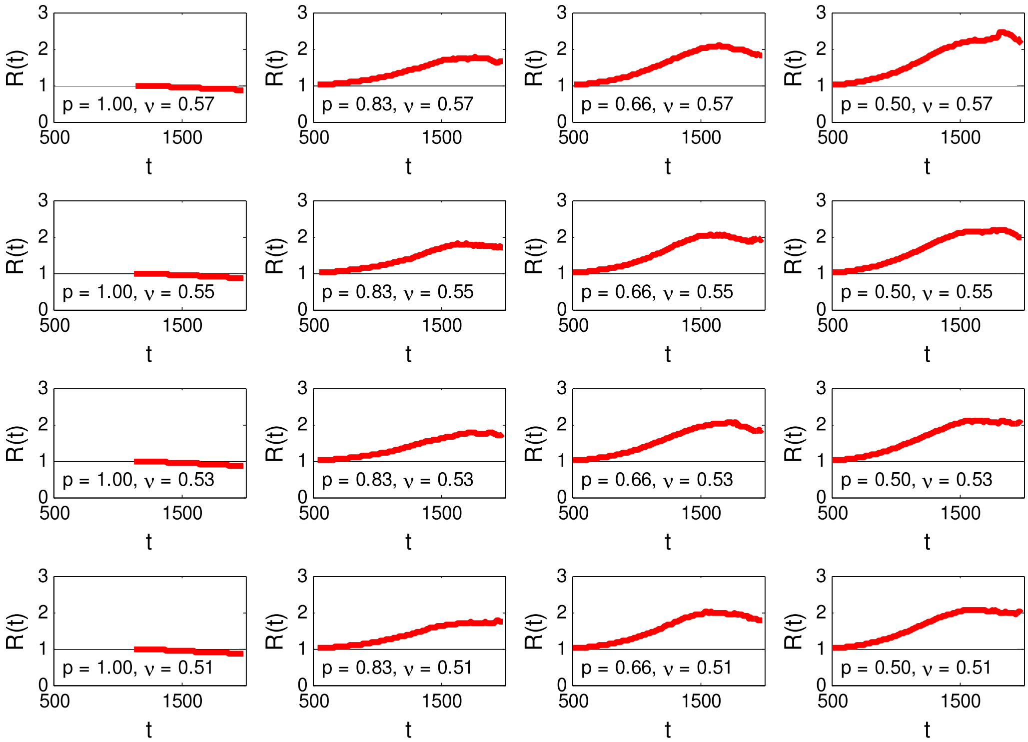

Fig. 3. Capacity OR with distractor predictions for a serial, self-terminating model. Each panel shows the results of a linear ballistic accumulator (Brown & Heathcote, 2008) simulation. For all simulations, the drift rate toward the target present boundary was fixed at 0.55 for the target source, but the drift rate, ν, of the distractor toward the target absent boundary was varied from the top row to the bottom from 0.57 to 0.51 in steps of 0.02. Hence, the top row shows the resilience function when distractors are processed faster than the target, and the bottom row shows the resilience function when distractors are processed slower than the targets. Processing was also varied from fixed-order, in the left-most column, to mixed-order in the remaining columns, with the probability of processing target AY before XB decreasing across the columns. The boundaries were set at 0.8, the range of the uniform start point distribution was 0.5, and the non-decision time was set to 0. Error accumulator drift rates were one minus the correct accumulator drift rates. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

because $S_{AY}^{serial}(t) = S_{AB}^{serial}(t)$ and $S_{AY}^{serial}(t) \ll S_{XB}^{serial}(t)$; hence, the double target AB_{OR} processing time from the serial model equals the minimum processing time of the two single targets from the serial model. Consequently, in the presence of distractors, a fixed-order serial self-terminating model will predict a resilience function that approaches $R(t) = 1$ as the distractor processing rate becomes slower than the processing rate of the relevant source (target). Furthermore, if processing is mixed-order (channel 1 is sometimes processed first, with probability p, and sometimes second), then the term on the left of Eq. (14) will be much greater than the term on the right, which could lead to $R(t) > 1$ depending on the distractor processing rate.

Fig. 3 demonstrates the $R(t)$ results of a simulation varying p, the probability that Channel 1 is processed first, at different levels of the distractor processing rate. Details of the simulation are provided in the figure caption. The figure displays a shift from fixed order ($p = 1$; channel 1 always processes first) to mixed order ($0 < p < 1$) as one moves from left to right across the columns. The thick red line marks the estimated $R(t)$ values. Distractor processing rate in the top row is less than the target processing rate. Specifically, the drift rate toward the correct boundary (i.e., in a serial linear ballistic accumulator model; Brown & Heathcote, 2008) was set at 0.57 for the distractors, and 0.55 for the single targets. As one moves down the rows, the drift rate of the distractors is reduced to 0.55 (second row; equal to the single targets), 0.53 (third row), and ultimately 0.51 (fourth row).

The interpretation of the results in Fig. 3 is straightforward; as processing shifts from fixed order to mixed order (along the horizontal axis), the estimated values of the resilience function increase regardless of the distractor processing rate (along the vertical axis). For completely fixed-order (left most column), $R(t) < 1$ when the processing of the distractors is faster than, or slightly less than the processing rate of single targets, but as processing of the distractors slows down, $R(t)$ values become closer to 1. For mixed-order serial processing, $R(t)$ is less than one, equal to one or greater than one depending on the rate of distractor processing. The take-home message is this: if distractors are present in the single-target trials and processing is serial and self-terminating, then $R(t)$ indexes the relative rate at which distractors and single targets are processed. It is worth noting, that since $R(t)$ and $C(t)$ are based on the same information, that unwary application of the capacity coefficient would lead to incorrect conclusions since, the value of the capacity coefficient is no longer necessarily limited even though processing is serial and self-terminating.

Coactive model with distractors. A coactive model pools in information from multiple sources into a single-evidence channel regardless of the type of evidence. For example, in Eidels et al. (2011), coactive processing was modeled by summing the evidence rates of two single targets, and in Fifić et al. (2010), coactive processing was modeled by assuming that the evidence accumulation rate was derived by integrating the joint bivariate distribution across both channels (i.e., stimulus dimensions) rather than the marginal distributions on each channel separately. This pooled processing produces responses to the double targets which are faster than the minimum time predictions of the benchmark UCIP model and results in $C_{OR}(t)$ values greater than 1.

In a coactive model, the evidence from the distractors is also assumed to be pooled into a common coactive channel, possibly combined with the information about targets when presented together. Like the serial model, the extent to which $R(t)$ is affected by distractors depends on how much evidence is provided by the distractors for the target and distractor responses. If the distractor provides strong evidence for the incorrect target

Table 1
Comparison of the OR capacity coefficient and resilience functions for each model.

Model	Without distractors	With distractors
Parallel self-terminating	$C_{OR}(t) = \frac{-\ln[S_{12}(t)]}{-\ln[S_1(t)\times S_2(t)]}$	$R(t) = \frac{-\ln[S_{AB}(t)]}{-\ln[S_{AY}(t)\times S_{XB}(t)]} = \frac{-\ln[S_{AB}(t)]}{-\ln[S_A(t)\times S_B(t)]}$
Serial self-terminating	$C_{OR}(t) = \frac{-\ln\left[1-\int_0^t (p[f_1(t)]+(1-p)[f_2(t)])dt\right]}{-\ln\left[\left(1-\int_0^t f_1(t)dt\right)\times\left(1-\int_0^t f_2(t)dt\right)\right]}$	$R(t) = \frac{-\ln\left[1-\int_0^t (p[f_A(t)]+(1-p)[f_B(t)])dt\right]}{-\ln\left[\left(1-\int_0^t (p[f_A(t)]+(1-p)[f_Y(t)*f_A(t)])dt\right)\times\left(1-\int_0^t (p[f_X(t)*f_B(t)]+(1-p)[f_B(t)])dt\right)\right]}$
Coactive	$C_{OR}(t) = \frac{-\ln[S_{12}(t)]}{-\ln[S_1(t)\times S_2(t)]}$	$R(t) = \frac{-\ln[S_{AB}(t)]}{-\ln[S_{AY}(t)\times S_{XB}(t)]}$
Parallel exhaustive	$C_{OR}(t) = \frac{-\ln[1-(F_1(t)\times F_2(t))]}{-\ln[(1-F_1(t))\times(1-F_2(t))]}$	$R(t) = \frac{-\ln[1-(F_A(t)\times F_B(t))]}{-\ln[(1-[F_A(t)\times F_Y(t)])\times(1-[F_X(t)\times F_B(t)])]}$
Serial exhaustive	$C_{OR}(t) = \frac{-\ln\left[1-\int_0^t (f_1(t)*f_2(t))dt\right]}{-\ln\left[\left(1-\int_0^t (f_1(t))dt\right)\times\left(1-\int_0^t (f_2(t))dt\right)\right]}$	$R(t) = \frac{-\ln\left[1-\int_0^t (f_A(t)*f_B(t))dt\right]}{-\ln\left[\left(1-\int_0^t [f_A(t)*f_Y(t)]dt\right)\times\left(1-\int_0^t [f_X(t)*f_B(t)]dt\right)\right]}$

Note: Processing channels for the single targets are labeled 1 and 2 for the models without distractors. Processing channels for the targets + distractors are labeled AY and XB for the models with distractors. For the serial self-terminating model, p is the probability of processing channel 1 before channel 2. For each model, we present each function in the form which most readily illustrates the key difference between the functions.

absent response, then the joint influence of fast double-target processing and slowed single target–single distractor processing will result in processing of the double targets that is much faster than the minimum processing time derived from the single target–single distractor cases. Furthermore, the degree to which the $R(t)$ function exceeds 1 depends on the discriminability of the distracting source. More salient distractors should have stronger and opposing processing rates than the target sources, slowing down processing and resulting in an even greater $R(t)$.

Parallel exhaustive model with distractors. In a parallel exhaustive model, all sources must be processed to completion regardless of whether the relevant source has finished processing. The cumulative density function of the double target is given by the maximum time distribution as follows:

$$F_{AB}(t) = F_A(t) \times F_B(t)\,. \tag{17}$$

Likewise for the single target–single distractor distributions, $F_{AY}(t) = F_A(t) \times F_Y(t)$ and $F_{XB}(t) = F_X(t) \times F_B(t)$. Consequently, the double target may be faster or slower than the minimum time derived from the single target–single distractor as each of the latter may be equivalent to the double target (i.e., because $F_{AB}(t) = F_{AY}(t) = F_{XB}(t)$ if the distractor is faster than the relevant, target source) or slower than the double target (i.e., if the distractor slows down single target–single distractor trials). Hence, $R(t)$ may be greater than or less than 1 depending on the relative rate of distractor processing compared to target processing.

Serial exhaustive model with distractors. In a serial exhaustive model, like a parallel exhaustive model, all sources of information must be processed regardless of whether the relevant source is processed first or not. The RT density for the double target is:

$$f_{AB}(t) = f_A(t) * f_B(t)\,. \tag{18}$$

Likewise for the single target–single distractor, $f_{AY}(t) = f_A(t) * f_y(t)$ and $f_{XB}(t) = f_X(t) * f_B(t)$. If the distractor is faster than the relevant (target) source then double-target responses may be slower than the derived minimum time of the single target–single distractor trials. On the other hand, if the distractor is slower than the target, then the double target may be faster than the derived minimum time, which implies that $R(t)$ may be greater than or less than 1 depending on the distractor processing rate.

Summary

The capacity coefficient function is affected by distractors and their relative processing rate. The above examples demonstrate our first major point: in the presence of distractors, the capacity coefficient (here termed resilience) cannot be simply interpreted as a measure of workload. Table 1 shows a comparison of the major differences between capacity and resilience for each of the models.

Our extension to the capacity coefficient can be thought of as containing the original redundant target design as a special

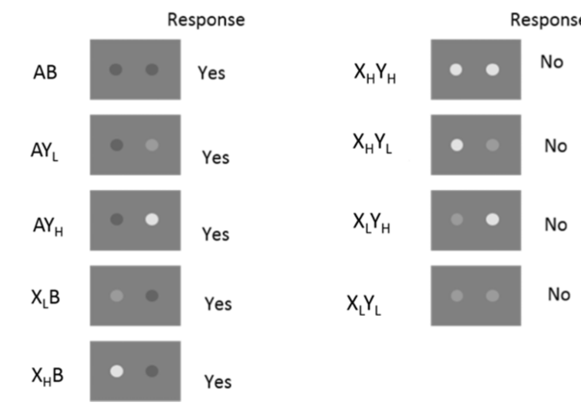

Fig. 4. Expanded discrimination task with varying levels of distractor discriminability.

case. When there is no target located in the non-target location, the processing times of these locations are presumably very short leading to negligible differences between the serial, parallel, and coactive single target cases without distractors. When distractors are present, however, our derivations show that the differences between these architectures change as the processing time of the non-target location increases (that is, as the non-target location attracts processing through the presence of distracting information).

With distractors, at best, the deviation of the function from $R(t) = 1$ can inform researchers that the system is not an unlimited, parallel self-terminating model. Other inferences are thwarted by the influence of the distractors on the derived minimum time. However, as we show next, this influence is systematic and can be used to regain the inferential power of the measure.

Resilience difference function

The fact that the $R(t)$ value of a serial self-terminating, coactive, serial exhaustive, and parallel exhaustive models depends on the relative processing rate of the distractors (compared with the target) suggests that the critical test for differentiating architectures is the change in the $R(t)$ function as the distractors become easier to process. To illustrate this point, we introduce a slight modification to the discrimination task introduced in Fig. 1 (right-hand side panel). We expand this task by adding distractors of different contrast in each location: one which is easy to discriminate and one which is hard to discriminate (see Fig. 4).

Let the subscripts L (low discriminability) and H (high discriminability) indicate distractors which are difficult and easy to discriminate from the target, respectively. For instance, consider the ease with which the distractor X_H can be rejected as not being a target (low contrast dot) compared to the ease with which

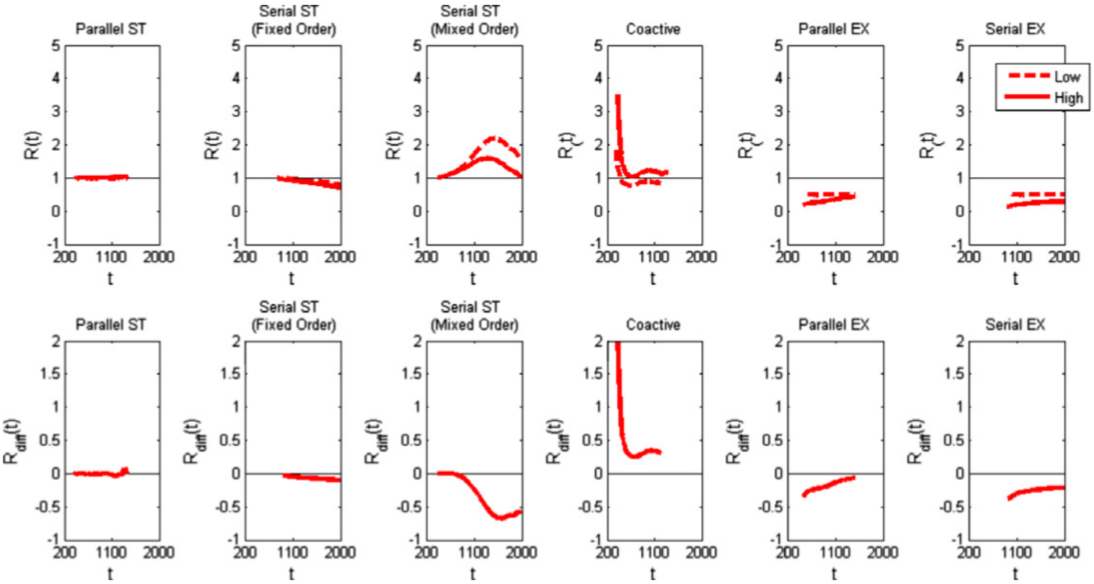

Fig. 5. Top row: $R(t)$ for each of the models computed from the low discriminability distractors (e.g., X_LB and AY_L) and the high discriminability distractors (e.g., X_HB and AY_H). Bottom row: $R_{diff}(t)$ for each of the models. The survivor functions which entered into the computation of $R(t)$ were generated from a linear ballistic accumulator (Brown & Heathcote, 2008) with the boundaries were set at 0.8, the range of the start point distribution was 0.5, and the non-decision time set to 0. The target drift rate and drift rate for the low discriminability distractor was set at 0.69; the high discriminability distractor drift rate was 0.94. Error accumulator drift rates were one minus the correct accumulator drift rates.

distractor X_L can be rejected as not being a target. The derived minimum time for the single target–single distractor containing high discriminability distractors (e.g., AY_H and X_HB) should be faster than the derived minimum time for the single target–single distractor containing low discriminability distractors (e.g., AY_L and X_LB). Hence, $R(t)$ for a serial model, to take one example, will be lower when computed with the high-discriminability distractors than the low-discriminability distractors.

In general, this ordering can be proved by starting from the two assumptions which underlie the interaction contrast and capacity coefficient measures of Systems Factorial Technology, namely, *selective influence* and *context invariance*. The resilience difference function requires selective influence, where the distribution functions (survivor function, in this case) of the high (H) and low (L) discriminability levels of an individual factor are ordered, such that $S_H(t) < S_L(t)$. Like the capacity coefficient, the $R(t)$ function further requires context invariance (Colonius, 1990), i.e., that the processing rate of a target does not change when presented with another source of information, e.g., $S_1(t) < S_{12}(t)$. Full details are provided in the Appendix, and predictions for each of the models are shown in Fig. 5.

Resilience difference function for a serial, self-terminating process. For a serial self-terminating model, depending on the order of serial processing (i.e., fixed-order or mixed-order), $S^{serial}_{AY_L}(t) \geq S^{serial}_{AY_H}(t)$ and $S^{serial}_{X_LB}(t) \geq S^{serial}_{X_HB}(t)$. Consequently, following from Eqs. (15) and (16) and Eq. (11), $R^{serial}_H(t) < R^{serial}_L(t)$, and hence:

$$R^{serial}_{diff}(t) = R^{serial}_H(t) - R^{serial}_L(t) \leq 0 \tag{19}$$

for all t.

Resilience difference function for a parallel, self-terminating process. The $R^{parallel}_{diff}(t)$ function stands in contrast to the $R^{serial}_{diff}(t)$ function because, as explained above, the discriminability value of the distractor does not matter, and hence:

$$R^{parallel}_{diff}(t) = R^{parallel}_H(t) - R^{parallel}_L(t) = 0 \tag{20}$$

for all t.

Resilience difference function for a coactive process. Under the assumption of context invariance, pooling a highly salient distractor with a target source should result in slower responding than

Table 2
Predictions for each model with and without distractors.

Model	Without distractors		With distractors	
	$C_{OR}(t)$	$C_{AND}(t)$	$R(t)$	$R_{diff}(t)$
Parallel self-terminating	$=1$		$=1$	$=0$
Serial self-terminating	<1		$<1, =1, >1$	<0
Coactive	>1	>1	>1	>0
Parallel exhaustive	<1	$=1$	$<1, =1, >1$	<0
Serial exhaustive	<1	<1	$<1, =1, >1$	<0

pooling a low-discriminability distractor with a target source of the same discriminability magnitude. Hence, $S^{serial}_{AY_L}(t) < S^{serial}_{AY_H}(t)$, $S^{serial}_{X_LB}(t) < S^{serial}_{X_HB}(t)$, and $R^{coactive}_H(t) > R^{coactive}_L(t)$ resulting in:

$$R^{coactive}_{diff}(t) = R^{coactive}_H(t) - R^{coactive}_L(t) > 0 \tag{21}$$

for all t.

Resilience difference function for a parallel and serial exhaustive processes. For both parallel and serial exhaustive models, the low discriminability distractor will slow down the single targets more than the high discriminability distractor. This means that for the low discriminability source the derived minimum time is slower than for the high discriminability sources, implying that $R^{parallel\,ex}_H(t) < R^{parallel\,ex}_L(t)$ and $R^{serial\,ex}_H(t) < R^{serial\,ex}_L(t)$. Hence,

$$R^{parallel\,ex}_{diff}(t) = R^{parallel\,ex}_H(t) - R^{parallel\,ex}_L(t) < 0 \tag{22}$$

and

$$R^{serial\,ex}_{diff}(t) = R^{serial\,ex}_H(t) - R^{serial\,ex}_L(t) < 0. \tag{23}$$

In summary, the resilience difference function, computed as the difference between the resilience functions with high- and low-discriminability distractors provides a novel qualitative contrast that distinguishes between serial, parallel, and coactive processing. Table 2 shows a comparison of the predictions for each of the models both with and without distractors.

This result echoes the mean RT predictions for the contrast category stimuli presented in Fifić et al. (2010); i.e., that a serial, self-terminating model predicts interior stimuli slower than exterior stimuli, a parallel, self-terminating model predicts equal

RTs for interior and exterior stimuli, and a coactive model predicts interior stimuli faster than exterior stimuli). Here we show that those mean RT predictions can be extended to a measure that accounts for the entire RT distribution.

The resilience results mirror the predictions of Townsend and Nozawa's (1995) capacity coefficient. However, we generalize their tool to a wider array of tasks that do not simply involve changes in workload but instead involve the presence of distractor information in the single target trials. Like existing measures of Systems Factorial Technology, the assumptions required are fairly minimal, namely effective selective influence, which can be verified by comparing the ordering high and low discriminability survivor functions, and context invariance (Townsend & Nozawa, 1995).

Discussion

In this paper, we extended an existing measure of workload capacity developed by Townsend and colleagues (Townsend & Nozawa, 1995; Townsend & Wenger, 2004) to examine the potential consequences of distractors presence in detection and discrimination tasks. This extension is critical for the interpretation of the capacity coefficients computed from a wide array of experiments in which distractor items could be present alongside targets. Using the standard capacity coefficient without accounting for distractors could limit (and even mislead) the researcher's ability to interpret empirical results, particularly if one considers prior research demonstrating clear links between architecture and capacity. For example, serial architectures predict limited capacity (when tested against a standard parallel model that forms the benchmark for $C(t)$ calculations; see Townsend & Nozawa, 1995); however, with distractors, the observed capacity may be unlimited or even supercapacity, even when the processing architecture is unambiguously identified as serial (see Fig. 3). Accounting for the inclusion of distractors in single-target displays allows for a more accurate interpretation of these results.

Relation to other nonparametric measures

Similar arguments as those presented here were used by Townsend and Nozawa (1997) to explain how a serial exhaustive model can violate the race model inequality. The race-model inequality (Miller, 1982) is an upper bound on performance with the double targets given RTs on single-target trials and a benchmark race model (which we termed UCIP model here). Violation of the inequality suggests that double targets are processed faster than can be expected by the parallel race model (and were taken by Miller to support coactivation). Townsend and Nozawa (1997) showed that if a serial model always processes both channels and the processing rates of the distractor channels are faster than the processing rates of the target channels then the serial model can violate the race model inequality. Using the recently developed synthesis of the race model inequality and workload capacity (Townsend & Eidels, 2011), our analysis shows that by slowing the derived minimum-time predictions through the inclusion of distracting information, any of the models we consider here except the baseline UCIP model can lead to $R(t) > 1$. Future work should consider whether these models also violate the race model inequality. The key difference between the design considered by Townsend and Nozawa (1997; akin to the discrimination design shown in Fig. 4) is that the distractor information is not uninformative. Instead, the distractor information provides evidence for the NO response (e.g., in Fig. 1), implying that simply ignoring the distractor information would be a non-optimal strategy.

Beyond understanding *why* capacity deviates from expectation when considering distractors, our analyses imbue the capacity measure with novel explanatory power. That is, rather than reflecting a simple change in workload, taking distractors into consideration allows the capacity measure of a serial model to index how fast distractors are processed relative to the single targets. Taken together with capacity, the resilience difference measure developed here provides further nonparametric constraints on the explanations of the empirical data.

The upshot of the current effort is this: it allows researchers to interpret the capacity coefficient measure in an expanded array of tasks, and reinforces the idea that capacity may be linked to architecture indirectly. Capacity *and* resilience should be considered along with architecture as a related but independent source of information about the processing system. Indeed, our analyses support the assertion that "these characteristics, although logically distinct, can interact in ways that can dupe or confound unwary researchers" (Townsend, Fific, & Neufeld, 2007). Townsend and colleagues' Systems Factorial Technology provides a framework for understanding these issues and our analyses augments that canon of work.

Relation to previous work on the capacity coefficient

As pointed out by Houpt et al. (2012), there are several ways in which performance on a double target condition might be compared to performance on a single target condition (either with or without distractors). Aside from the standard calculation of capacity measure, $C(t)$, and the resilience measure, $R(t)$, proposed and analyzed in this paper, other capacity variations might be computed by comparing the double-target performance to only one of the single targets (with and without a distractor) or by comparing performance on a single target with a distractor to performance on a single target without a distractor. Each of these cases requires a different set of conditions which may or may not be able to be adequately instantiated in a particular experimental paradigm. For instance, the task shown in Fig. 4 precludes examining the standard capacity measure; nonetheless, our new resilience measure provides valuable information about the processing system. We suspect that combining a number of these capacity variations will provide a large array of nonparametric results, which can then be used to constrain theories of information processing. Moreover, various capacity measures may differentiate different processes and allow intimate understanding of more specific processing components (such as, for instance, the effects of distractors vs. load).

Conclusion

Standard measures of workload capacity (e.g., Townsend & Nozawa, 1995) describe how human performance changes with increase in processing load. We derived a new measure, $R(t)$, that augments existing measures by taking into account contextual factors, such as the presence vs. absence of distractors, and their relative processing rate. This theoretical development constrains the interpretation of empirical data and allows better understanding of information processing under varying load conditions. Future work can examine the consequences of relaxing certain assumptions, such as the assumption of error free performance, the assumption of context invariance, and develop statistical methods for testing $R(t)$ and $R_{diff}(t)$ analogous to Houpt and Townsend's (2012) work on the statistics for the capacity coefficient. Future work can also relate mathematically the $R(t)$ and $R_{diff}(t)$ with other measure of human performance and develop bounds analogous to the upper and lower limits on the unlimited capacity predictions of the UCIP model (Townsend & Eidels, 2011).

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Appendix. Resilience proofs

It is useful to recall that if $S_1(t) < S_2(t)$ then $-\ln(S_1(t)) > -\ln(S_2(t))$ and, hence, $\frac{1}{-\ln(S_1(t))} < \frac{1}{-\ln(S_2(t))}$.

The key identity is the prediction that the processing time of a double target in a parallel processing system should equal the minimum time derived from the processing times of two single targets (i.e. assuming that responding can terminate as soon as the target is detected—i.e., an OR task):

$$S_{12}(t) = S_1(t) \times S_2(2). \tag{A.1}$$

In the present formulation, there is distractor information present during the previously single target trials; however, in an unlimited capacity, independent parallel self-terminating model, this information does not affect the identity:

$$S_{AB}(t) = S_{AY}(t) \times S_{XB}(t). \tag{A.2}$$

Using the first identity one can compute a baseline measure of capacity as follows:

$$C(t) = \frac{-\ln[S_{12}(t)]}{-\ln[S_1(t) \times S_2(t)]}. \tag{A.3}$$

Using the second identity, one can compute a baseline measure of resilience as:

$$R(t) = \frac{-\ln[S_{AB}(t)]}{-\ln[S_{AY}(t) \times S_{XB}(t)]}. \tag{A.4}$$

These two functions require taking the negative logarithm of each side of their respective equations. This transforms the Survivor function form into an integrated Hazard function (Luce, 1986; Townsend & Ashby, 1983 and Townsend & Nozawa, 1995), which means that under unlimited capacity, independent parallel, self-terminating processing $C(t)$ and $R(t)$ equal 1 for all t. This transformation also implies that any double target that is slower than the derived minimum time will result in a $C(t)$ or $R(t)$ function which is less than 1, respectively. By contrast, any double target which is faster than the derived minimum time result in a $C(t)$ or $R(t)$ function which is greater than 1, respectively.

The key to understanding resilience is to note that for architectures other than the parallel architecture, adding distractor information can slow down the derived minimum time thereby potentially allowing the double target to be faster than the derived minimum time and resulting in an $R(t)$ function greater than 1. The more distractor information slows down the minimum time, the larger the value of the $R(t)$ function. We next prove this intuitive idea for the serial model.

Serial, self-terminating model. The RT density of a serial, self-terminating model for the double target is given by

$$f_{AB}^{serial}(t) = p[f_A(t)] + (1-p)[f_B(t)]. \tag{A.5}$$

For the target + distractor conditions, the RT densities are:

$$f_{AY}^{serial}(t) = p[f_A(t)] + (1-p)[f_Y(t) * f_A(t)] \tag{A.6}$$

and

$$f_{XB}^{serial}(t) = p\,[f_X(t) * f_B(t)] + (1-p)[f_B(t)]. \tag{A.7}$$

Substituting these equations into the minimum time identity gives:

$$\begin{aligned} &S_{AB}^{serial}(t) \\ &= 1 - \int_0^t (p[f_A(t)] + (1-p)[f_B(t)])\,dt \\ &= \left[1 - \int_0^t (p[f_A(t)] + (1-p)[f_X(t) * f_B(t)])\,dt\right] \\ &\quad \times \left[1 - \int_0^t (p\,[f_A(t) * f_Y(t)] + (1-p)[f_B(t)])\,dt\right] \\ &= S_{AY}^{serial}(t) \times S_{XB}^{serial}(t). \end{aligned} \tag{A.8}$$

Proposition 1. *The $R(t)$ function can equal 1 for fixed-order serial, self-terminating processing.*

Proof 1. The proposition is equivalent to assuming that the serial double target is faster than the derived minimum time of the incongruent targets, i.e., $S_{AB}^{serial}(t) \le S_{AY}^{serial}(t) \times S_{XB}^{serial}(t)$.

If processing occurs in a fixed-order such that channel 1, containing A or X, is always processed before channel 2, containing B or Y (i.e., $p = 1$), then:

$$\begin{aligned} S_A^{serial}(t) &= 1 - \int_0^t (f_A(t))\,dt \\ &= \left[1 - \int_0^t (f_A(t))\,dt\right] \times \left[1 - \int_0^t (f_X(t) * f_B(t))\,dt\right] \\ &= S_A^{serial}(t) \times S_{XB}^{serial}(t). \end{aligned} \tag{A.9}$$

If $S_A^{serial}(t) \ll S_{XB}^{serial}(t)$ (i.e., XB is much slower due to the processing time of X such that A has no probability of finishing after XB), then

$$S_A^{serial}(t) = S_A^{serial}(t) \times S_{XB}^{serial}(t) = S_A^{serial}(t);\ \text{hence,}\ R(t) = 1.$$

Proposition 2. *The $R(t)$ function can exceed 1 for mixed-order serial, self-terminating processing.*

Proof 2. For a mixed-order serial, self-terminating process, proving the $R(t) \ge 1$ requires proving that

$$\begin{aligned} S_{AB}^{serial}(t) &\le \left[1 - \int_0^t (p[f_A(t)] + (1-p)[f_X(t) * f_B(t)])\,dt\right] \\ &\quad \times \left[1 - \int_0^t (p[f_A(t) * f_Y(t)] + (1-p)[f_B(t)])\,dt\right]. \end{aligned}$$

It should be clear that if X and Y are much slower than A and B, then this inequality will hold. □

Proposition 3. *Under a serial, self-terminating process, if $S_{Y_H}(t) < S_{Y_L}(t)$, $S_{X_H}(t) < S_{X_L}(t)$ and assuming context invariance, then $R_H(t) < R_L(t)$.*

Proof 3. Proving this result requires proving that the derived minimum time is slower when the incongruent targets are low discriminability than when they are high discriminability. Recall that the slower the minimum time, the smaller the denominator in the $R(t)$ function. Hence, we need to show that:

$$S_{AY_L}(t) \times S_{X_LB}(t) > S_{AY_H}(t) \times S_{X_HB}(t) \tag{A.10}$$

which expands to:

$$\left[1-\int_0^t \left(p\left[f_A(t)\right]+(1-p)\left[f_{X_L}(t)*f_B(t)\right]\right)dt\right] \times\left[1-\int_0^t \left(p\left[f_A(t)*f_{Y_L}(t)\right]+(1-p)\left[f_B(t)\right]\right)dt\right] > \left[1-\int_0^t \left(p\left[f_A(t)\right]+(1-p)\left[f_{X_H}(t)*f_B(t)\right]\right)dt\right] \times\left[1-\int_0^t \left(p\left[f_A(t)*f_{Y_H}(t)\right]+(1-p)\left[f_B(t)\right]\right)dt\right]. \quad (A.11)$$

Based on the assumption of selective influence, we assume that $S_{Y_H}(t) < S_{Y_L}(t)$ and $S_{X_H}(t) < S_{X_L}(t)$; therefore, $\left[f_A(t)*f_{Y_L}(t)\right] > \left[f_A(t)*f_{Y_H}(t)\right]$ and $\left[f_{X_L}(t)*f_B(t)\right] > \left[f_{X_H}(t)*f_B(t)\right]$. Substitution into (A.11) proves the inequality. □

Proposition 4. *For a serial, self-terminating model,* $R_{diff}(t) = R_H(t) - R_L(t) < 0$ *for all* t.

Proof 4. The proof follows directly from Proof 3. □

Parallel, self-terminating model.

Proposition 5. *For a parallel, self-terminating model,* $R_{diff}(t) = R_H(t) - R_L(t) = 0$ *for all* t.

Proof 5. The proof follows directly from the assumption that $S_{AB}(t) = S_{AY}(t) \times S_{XB}(t) = S_A(t) \times S_B(t)$. □

Coactive model. For a coactive model, we want to prove that $R_{diff}(t) = R_H(t) - R_L(t) > 0$ because $R_H(t) > R_L(t)$. This proof is difficult because it depends on particular parameterizations of the coactive model. Recall that the capacity for a coactive model has only been proven for a Poisson counter model (Townsend & Nozawa, 1995) and a Weiner diffusion model (Houpt & Townsend, 2012). The same problem applies here.

However, the intuition is that the rate of processing will be slowed down more by high discriminability target than by a low discriminability target; consequently, the derived minimum time will be faster for low discriminability than for high discriminability targets: $S_{AY_L}(t) \times S_{X_LB}(t) < S_{AY_H}(t) \times S_{X_HB}(t)$.

Consequently, $R_H(t) > R_L(t)$.

Parallel exhaustive model. In a parallel exhaustive model, all sources must be processed to completion regardless of whether the target source has finished processing. The cumulative density function of the double target is given by the maximum time distribution as follows:

$$F_{AB}(t) = F_A(t) \times F_B(t). \quad (A.12)$$

Likewise for the target + distractors, $F_{AY}(t) = F_A(t) \times F_Y(t)$ and $F_{XB}(t) = F_X(t) \times F_B(t)$. Consequently, the double target may be faster or slower than the derived minimum time of the target + distractors as each of the target + distractor may be equivalent to the double target (i.e., because $F_{AB}(t) = F_{AY}(t) = F_{XB}(t)$ if the distractor source is faster than the target source) or slower than the double target (i.e., if the distractor source slows down the target + distractors). Hence, $R(t)$ may be greater than or less than 1 depending on the incongruent source processing rate.

Serial exhaustive model. In a serial exhaustive model, like a parallel exhaustive model, all sources must be processed regardless of whether the target source is processed first or not. The RT density for the congruent target is:

$$f_{AB}(t) = f_A(t) * f_B(t) \quad (A.13)$$

where the $*$ symbol indicates the convolution of the two target densities. Likewise for the incongruent targets, $f_{AY}(t) = f_A(t) * f_Y(t)$ and $f_{XB}(t) = f_X(t) * f_B(t)$. If the incongruent source is faster than the target source then the congruent target may be slower than the derived minimum time from the single targets. On the other hand, if the incongruent source is slower than the target source, then the congruent target may be faster than the derived minimum time, which implies that $R(t)$ may be greater than or less than 1 depending on the incongruent source processing rate.

For the parallel and serial exhaustive models, the low discriminability distractor will slow down the targets more than the high discriminability distractor. This will mean that for the low discriminability source the derived minimum time is slower than for the high discriminability sources implying that $R_H^{parallel\,ex}(t) < R_L^{parallel\,ex}(t)$ and $R_H^{serial\,ex}(t) < R_L^{serial\,ex}(t)$. Hence,

$$R_{diff}^{parallel\,ex}(t) = R_H^{parallel\,ex}(t) - R_L^{parallel\,ex}(t) < 0 \quad (A.14)$$

and

$$R_{diff}^{serial\,ex}(t) = R_H^{serial\,ex}(t) - R_L^{serial\,ex}(t) < 0. \quad (A.15)$$

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